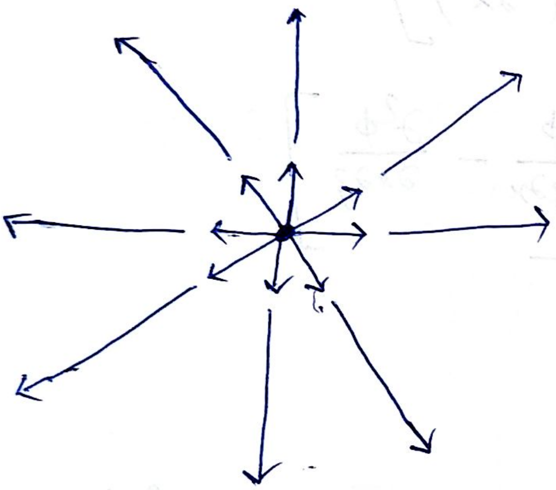
Dr. S.K. Yadav; email: sunilophy30@gmail.com

Geometrical Interpretation of divergence and Curl:-

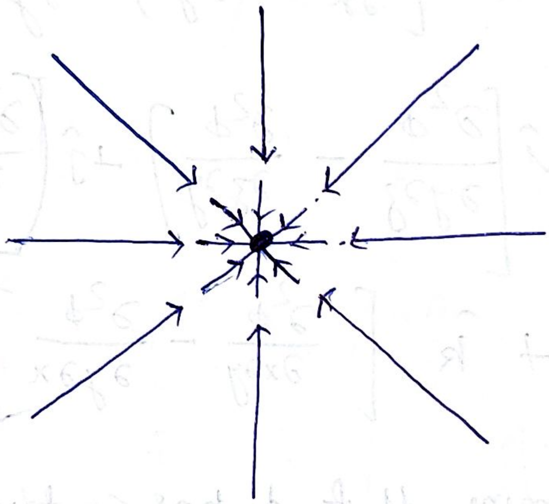
Divergence — $\boxed{\cancel{\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}}$

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}, \quad \text{where} \quad \vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$

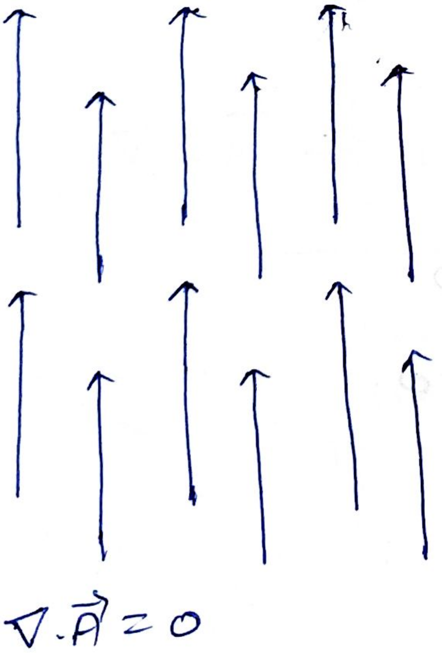
$\nabla\cdot\vec{A} \rightarrow$ measure of how much the vector $\vec{A}$ diverges or spreads out from the particular point in the Question



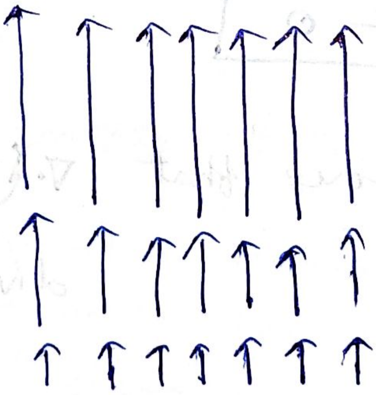
$\nabla\cdot\vec{A} = +\text{ive}$



$\nabla\cdot\vec{A} = -\text{ive}$



$\nabla\cdot\vec{A} = 0$

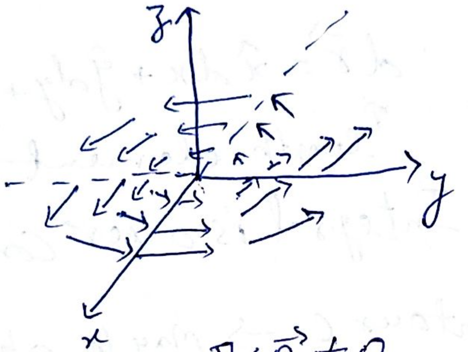


$\nabla\cdot\vec{A} = +\text{ive}$

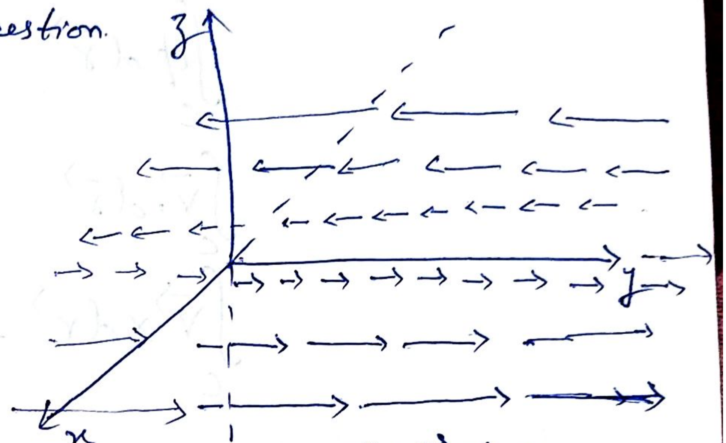
Curl: $$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$\nabla\times\vec{A} \rightarrow$ measure of how much vector $\vec{A}$ curls around the particular point in question.



$\nabla\times\vec{A} \neq 0$



$\nabla\times\vec{A} \neq 0$

In Figures on page (1) $\nabla\times\vec{A} = 0$.

Some Identities:

(i) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(ii) $\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\nabla)\vec{A}$

(iii) $\nabla\cdot(f\vec{A}) = f(\nabla\cdot\vec{A}) + \vec{A}\cdot(\nabla f)$

(iv) $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$

(v) $\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) - \vec{A}\times(\nabla f)$

(vi) $\nabla\cdot\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A})$

(vii) $\nabla\cdot(\nabla\times\vec{A}) = 0$

(viii) $\nabla\times(\nabla f) = 0$

(ix) $\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2 A$

H.W. Take vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ and verify above identities.

$A_x, A_y, A_z \mid B_x, B_y\ B_z$ all are functions of (x, y, z)